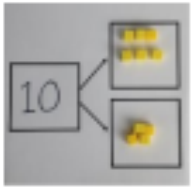
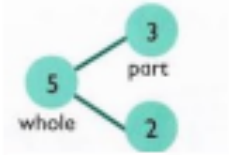
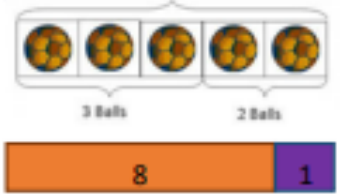
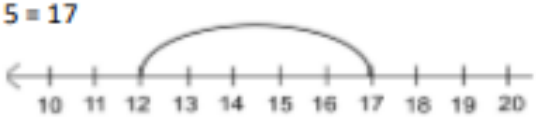
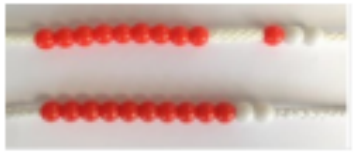
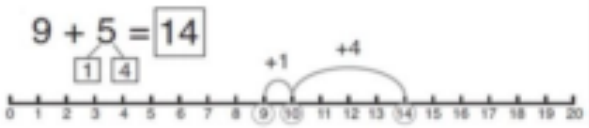
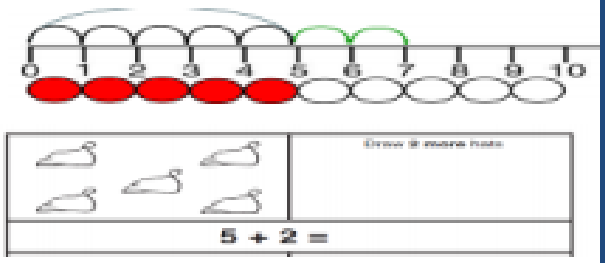
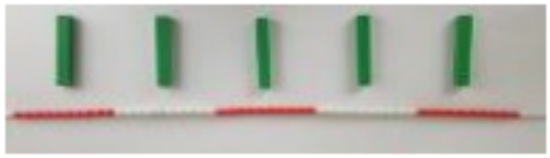
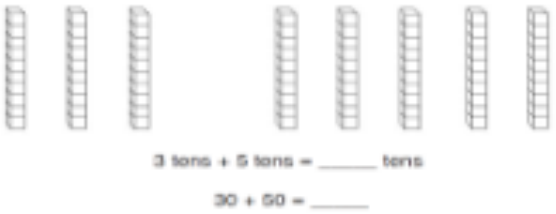
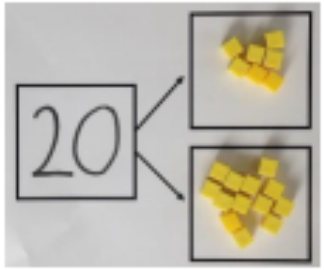
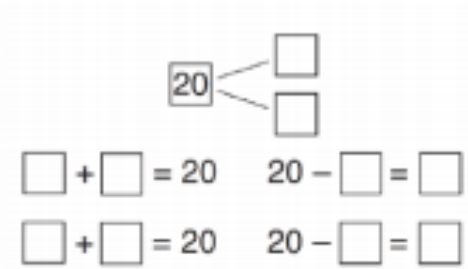
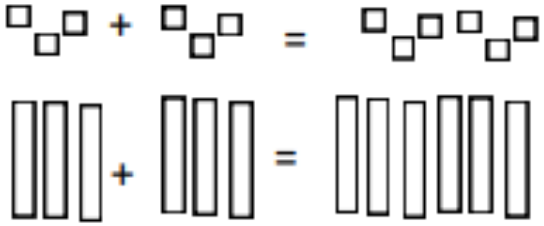
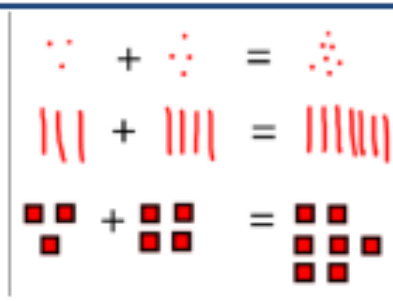
Addition

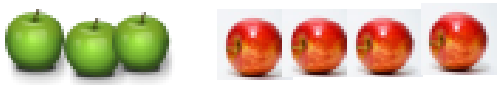
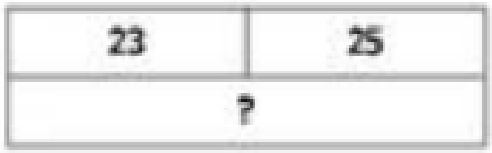
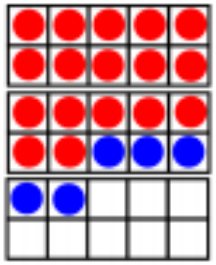
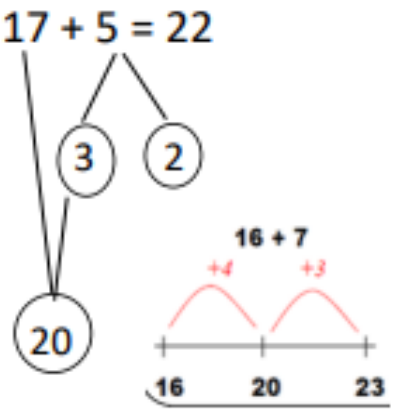
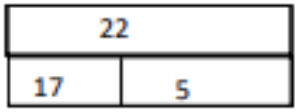
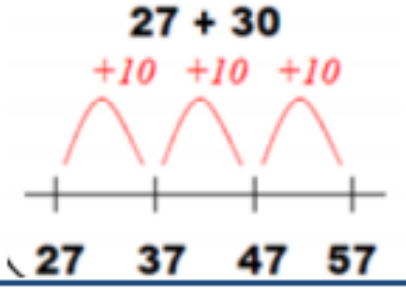
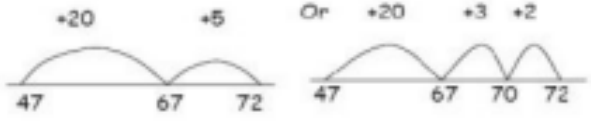
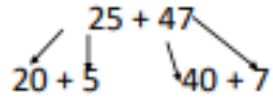
Key Vocabulary: sum, total, parts and whole, plus, add, altogether, more, count on, 'is equal to', 'is the same as'

Objective & Strategy	Concrete	Pictorial	Abstract
Combine two parts to make a whole: part-whole model	   Use part part whole model. Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar.	$4 + 3 = 7$  $10 = 6 + 4$ Use the part-part whole diagram as shown above to move into the abstract.
Starting at the big number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	$12 + 5 = 17$  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	$5 + 12 = 17$ Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later*	 $6 + 5 = 11$  Start with the bigger number and use the smaller number to make 10. Use ten frames.	 Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 	$7 + 4 = 11$ If I am at seven, how many more do I need to make 10. How many more do I add on now?

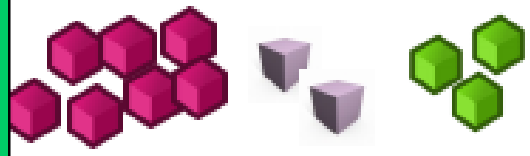
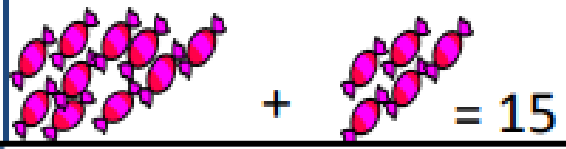
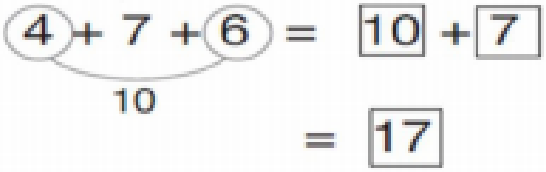
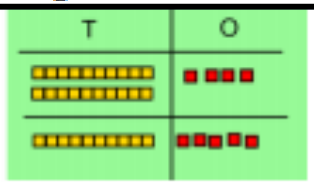
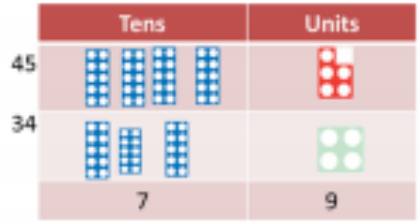
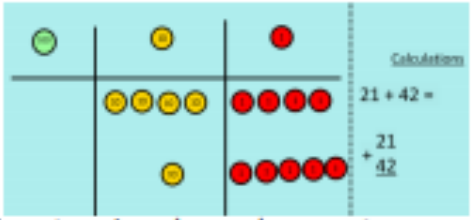
Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Represent & use number bonds and related subtraction facts within 100.	 2 more than 5.		Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*
Adding multiples of ten.	50= 30 = 20  Model using dienes and bead strings	 Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part whole model	 Children explore ways of making numbers within 20		□ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Using known facts		 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700

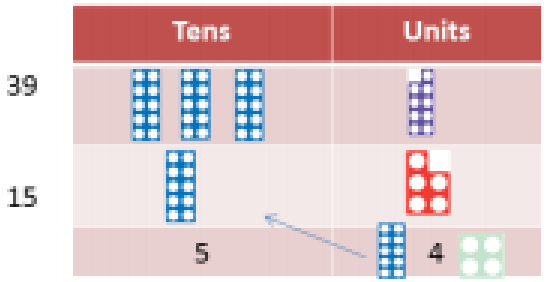
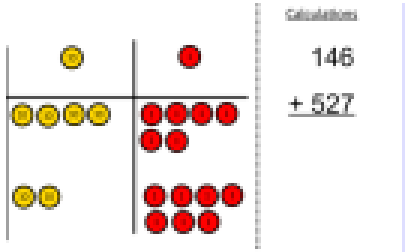
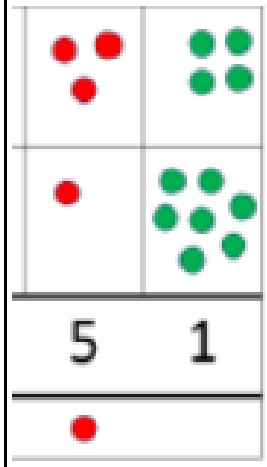
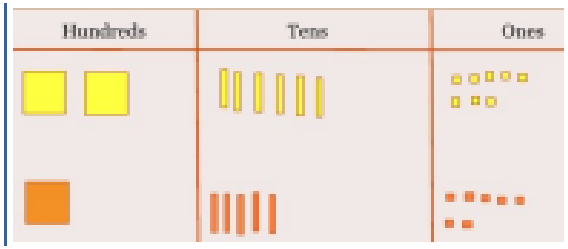
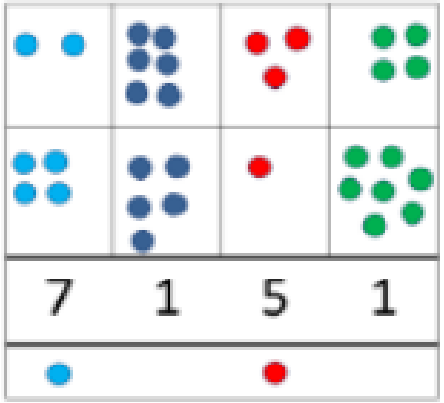
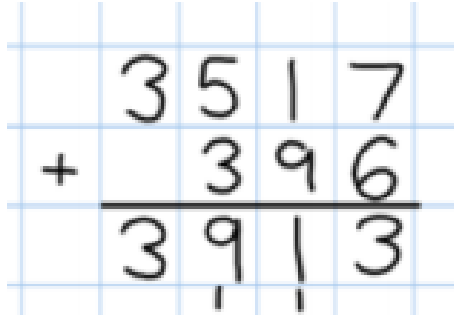
Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Bar Model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48
Add a two digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part part whole and number line to model. 	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
Add a two digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change		27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2-digit numbers	 Model using dienes , place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	 20 + 40 = 60 5+ 7 =12 60 + 12 = 72

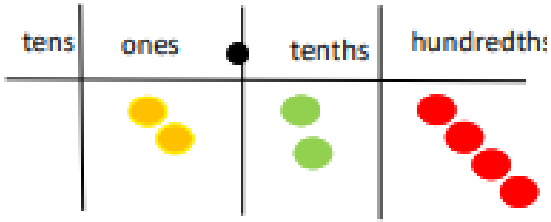
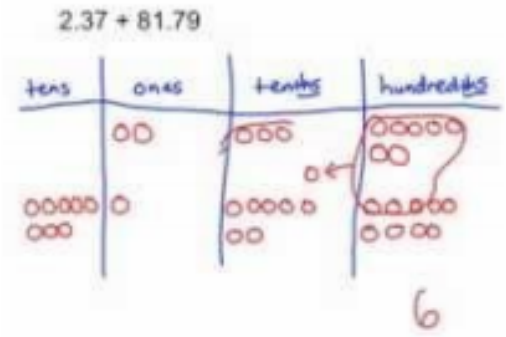
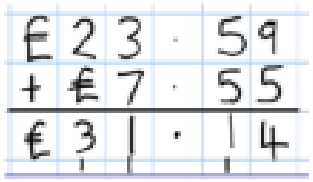
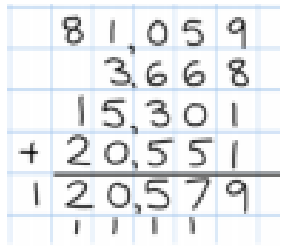
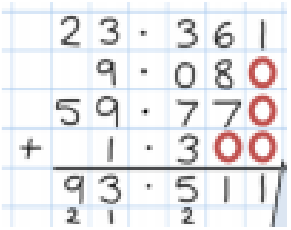
Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit	 Regroup and draw representation. 	 Combine the two numbers that make/ bridge ten then add on the third.
Column Addition—no regrouping Add two or three 2 or 3 digit numbers	 Model using Dienes or nu-micon Add together the ones first, then the tens.   Move to using place value counters	Children move to drawing the counters using a tens and one frame. 	2 2 3 + 1 1 4 3 3 7 Add the ones first, then the tens, then the hundreds.

Addition

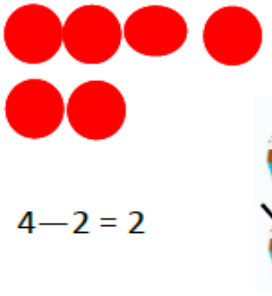
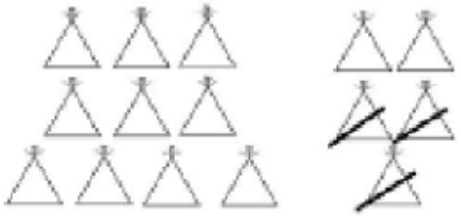
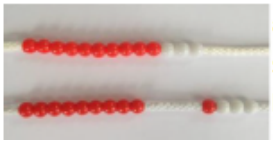
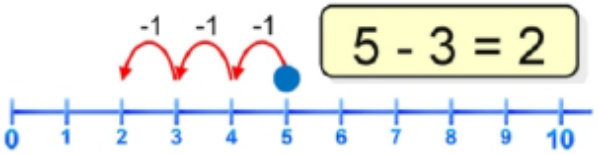
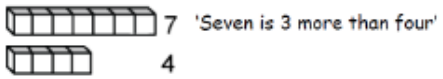
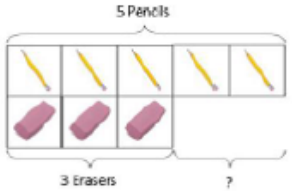
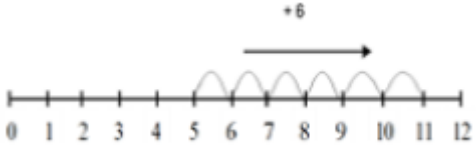
Objective & Strategy	Concrete	Pictorial	Abstract
Column Addition with regrouping	 Exchange ten ones for a ten. Model using numicon and pv counters. 	 Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line	$20 + 5$ $40 + 8$ $60 + 13 = 73$ Start by partitioning the numbers before formal column to show the exchange. 536 + 85 621 1 1
Add numbers with up to 4 digits	Children continue to use Dienes or pv counters to add, regrouping ten ones for a ten, ten tens for a hundred and ten hundreds for a thousand. 	Drawn representations using pv grid 	Continue from previous work to carry hundreds as well as tens Relate to money and measures. 

Addition

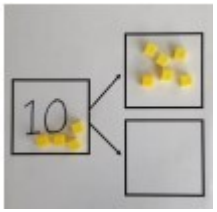
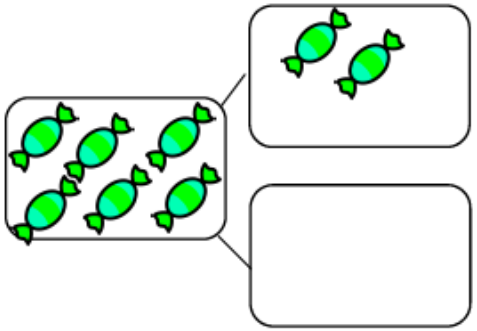
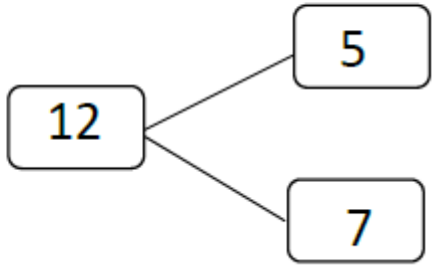
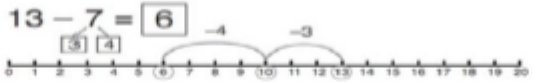
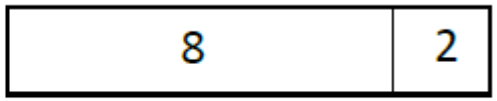
Objective & Strategy	Concrete	Pictorial	Abstract
Add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	Introduce decimal place value counters and model regrouping for addition. 		 
Add several numbers of increasing complexity. Including adding money, measure and decimals with different numbers of decimal points.	As Above	As Above	Insert zeros for place value holders.  

Subtraction

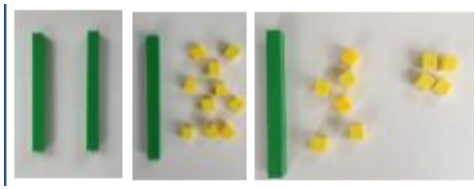
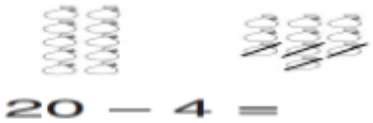
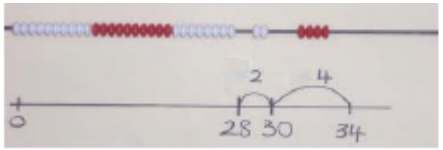
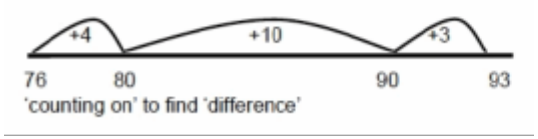
Key Vocabulary: take away, less than, the difference, subtract, minus, fewer, regroup

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters , cubes etc to show how objects can be taken away.  6—4 = 2 4—2 = 2	 15 – 3 = 12 Cross out drawn objects to show what has been taken away.	7—4 = 3 16—9 = 7
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the difference	Compare objects and amounts  Lay objects to represent bar model 	Count on using a number line to find the difference 	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?

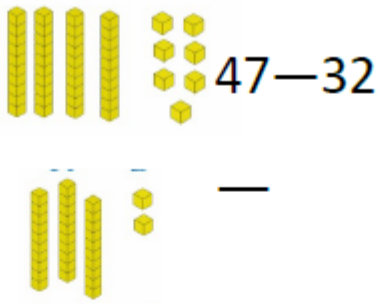
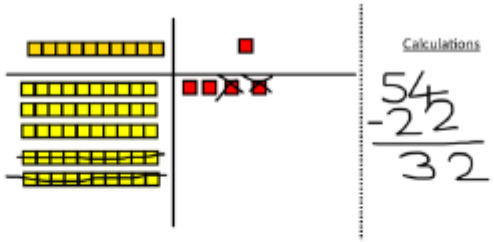
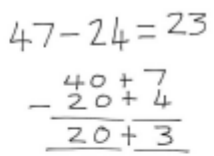
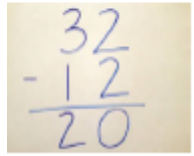
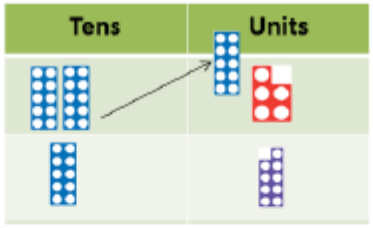
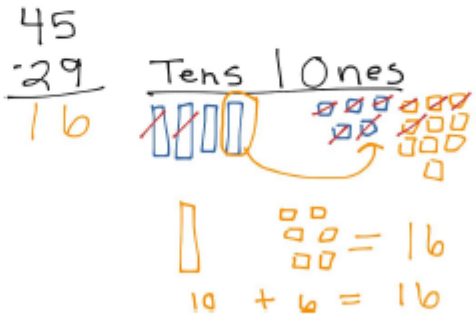
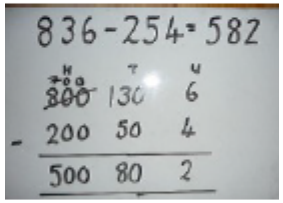
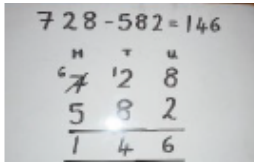
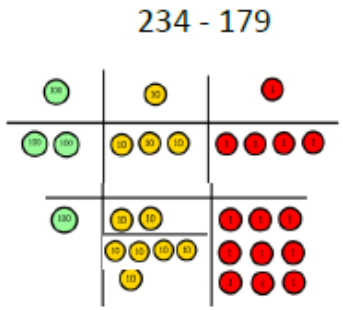
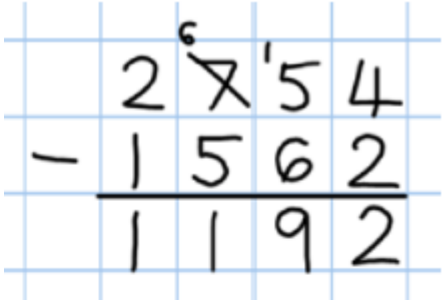
Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20	 Link to addition. Use Part Part Whole model to model the inverse. If 10 is the whole and 6 is one of the parts, what is the other part? 10—6 = 4	Use pictorial representations to show the part. 	Move to using numbers within the part whole model. 
Make 10	Make 14 on a ten frame. Take 4 away to make ten, then take one more away so that you have taken 5. 14 — 9 	Jump back 3 first, then another 4. Use ten as the stopping point. 13—7 	16 — 8 How many do we take off first to get to 10? How many left to take?
Bar Model	 5—2 = 3		 10 = 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2

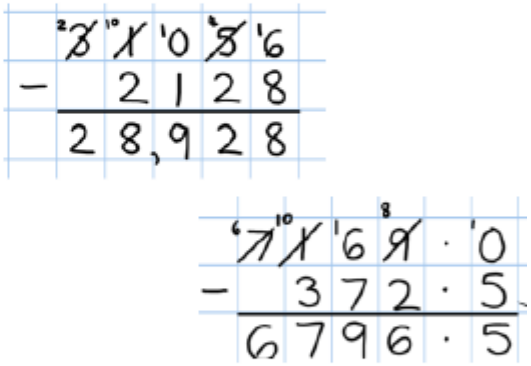
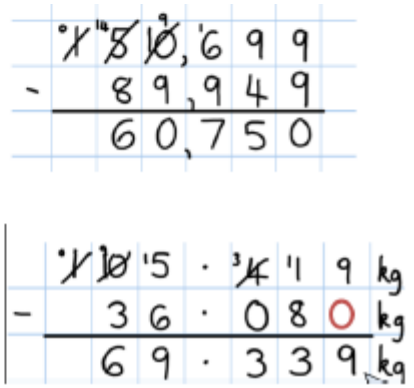
Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	Use a Place value chart to show how to regroup a ten into ten ones. 	 	20 — 4 = 16
Partitioning to subtract without regrouping	34 — 13 = 21 Use Dienes to show how to partition the number when subtracting without regrouping. 	Children draw representations of Dienes and cross off  43—21 = 22	43 — 21 = 22
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds*	Use a bead string or number line to model counting on to the next ten and the rest  34—28	Use a number line to count on to next ten and then the rest  	93 — 76 = 17

Subtraction

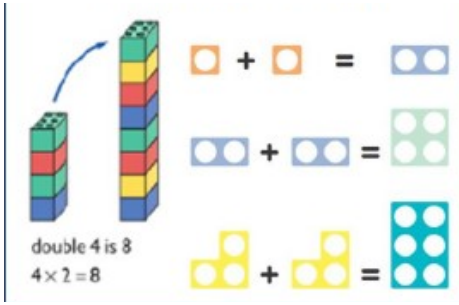
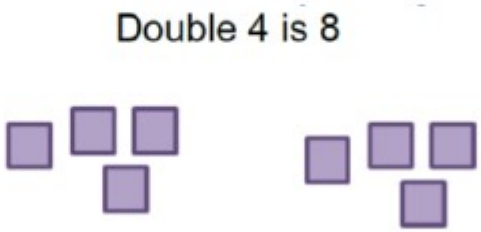
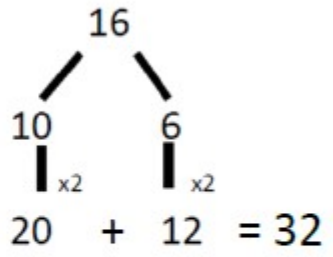
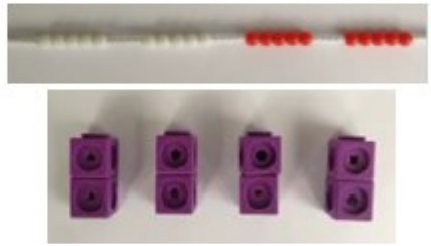
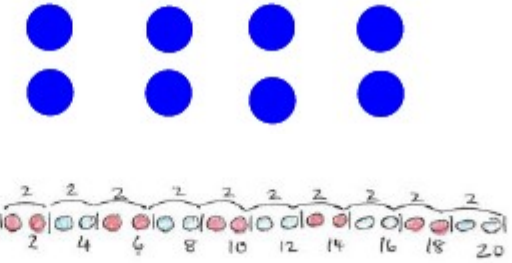
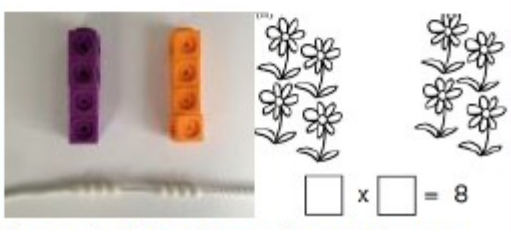
Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping	Use base 10 or Numicon to model 	Draw representations to support understanding 	Intermediate step may be needed to lead to clear subtraction understanding  
Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to PV counters, modelling the regroup of a ten into ten ones.	 Children may draw base ten or PV counters and cross off	Begin by partitioning into PV columns  Then move to formal method 
Subtracting tens and ones (up to 4 digit numbers) *Introduce decimal subtraction through context of money*	Model process of regrouping using Numicon, base ten and then move to PV counters 	Children draw PV counters and show their regrouping	

Subtraction

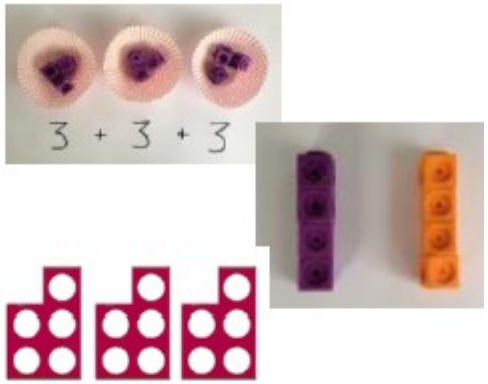
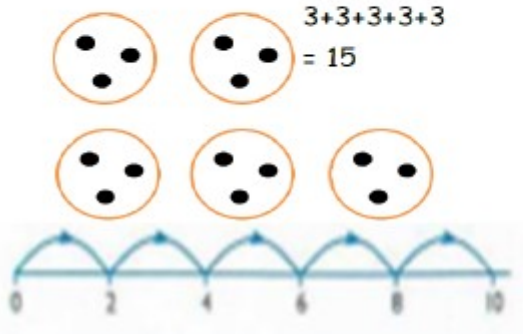
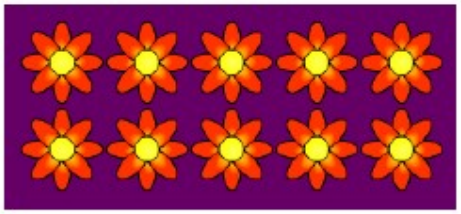
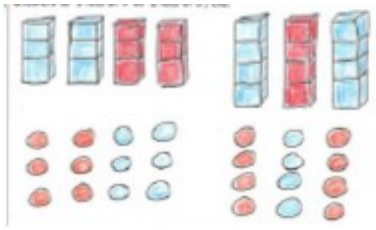
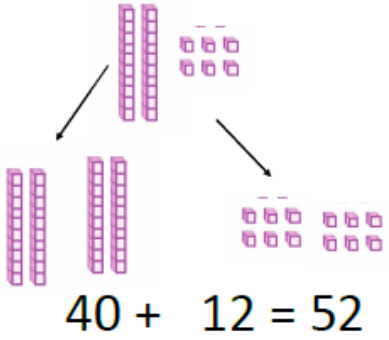
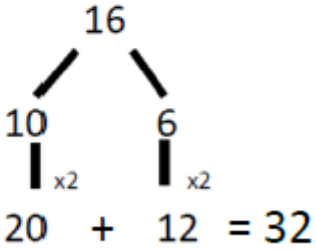
Objective & Strategy	Concrete	Pictorial	Abstract
Subtract with at least 4 digits, including money and measures *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal*		Children to draw PV counters and show their regrouping.	Use zeros for place value holders 
Subtract with increasingly large and more complex numbers and decimals values.			

Multiplication

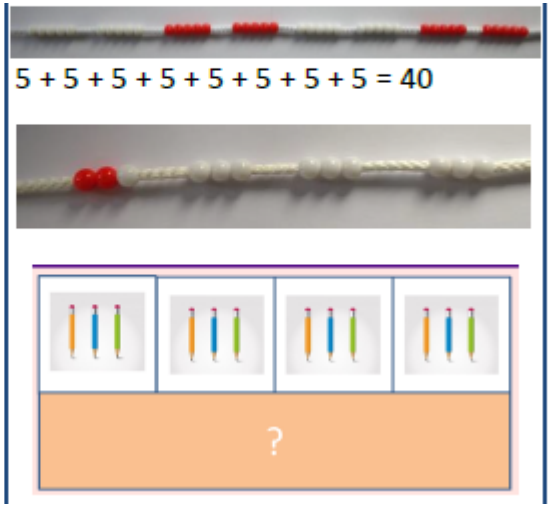
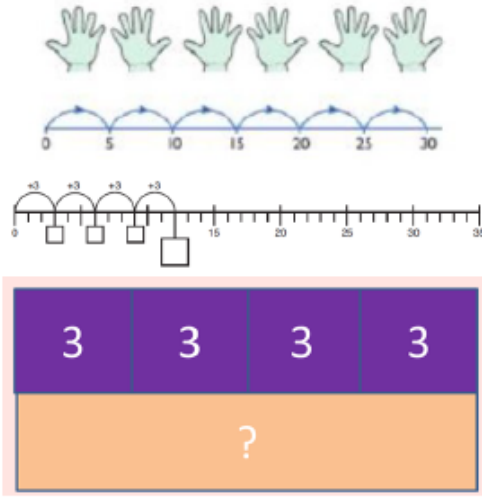
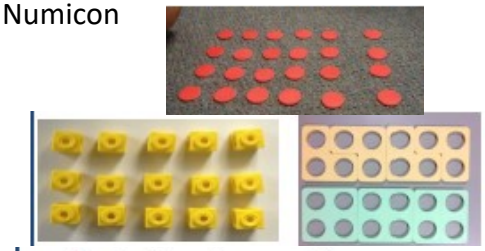
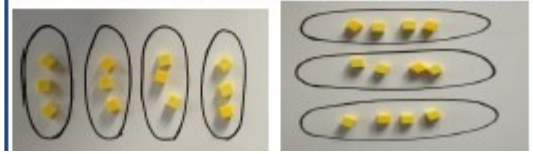
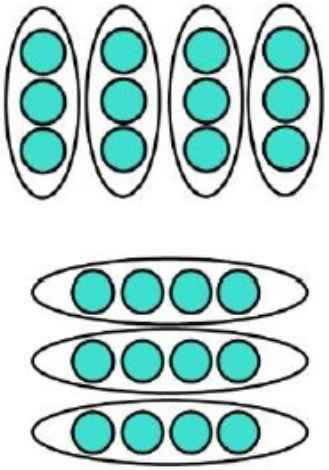
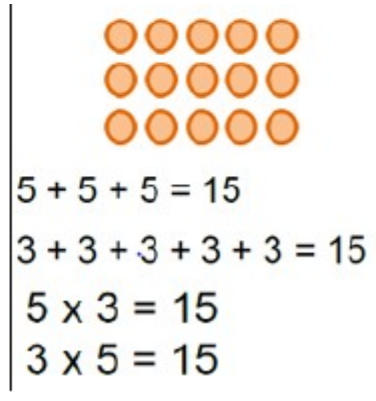
Key Vocabulary: double, times, multiplied by, the product of, groups of, lots of, equal groups

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling. 	Draw pictures to show how to double numbers. 	Partition a number and then double each part before recombining it back together. 
Counting in multiples	Count the groups as children are skip counting. Children may use their fingers as they are skip counting (2 4 6 8). 	Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
Making equal groups and counting	Use manipulatives to create equal groups. 	Draw  to show 2 X 3 = 6 Draw and make representation.	2 X 4 = 8

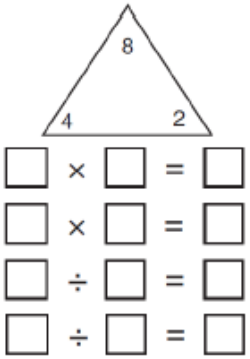
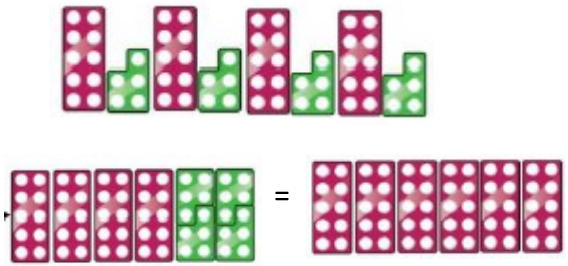
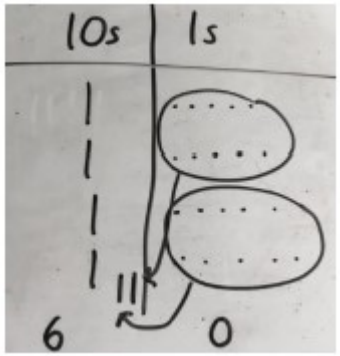
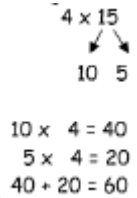
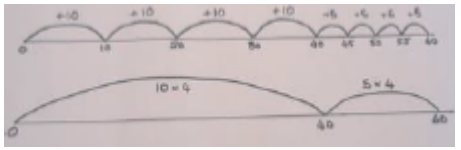
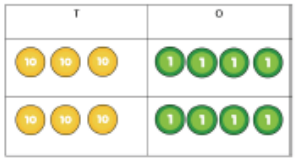
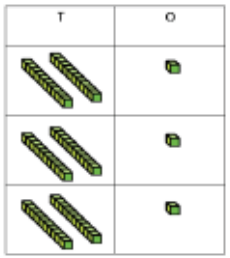
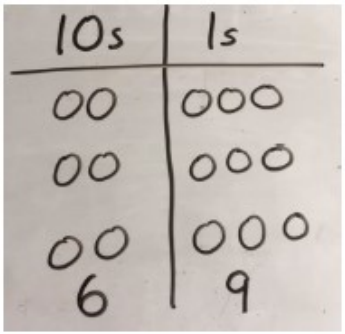
Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	Use different objects to add equal groups 	Use pictures including number lines to solve problems *There are 3 sweets in a bag. How many sweets are there in 5 bags?* 	Write addition sentences to describe objects and pictures 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots of 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	3 X 2 = 6 2 X 5 = 10
Doubling	Model doubling using Dienes and PV counters 	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 

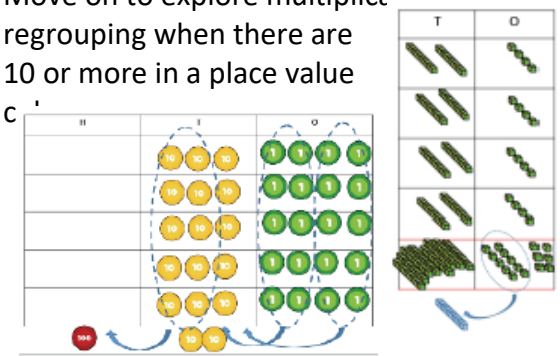
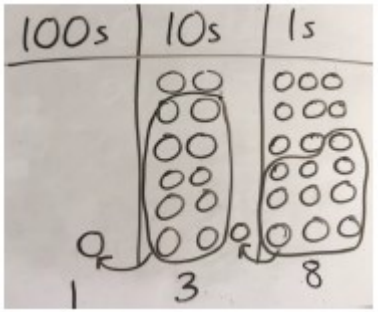
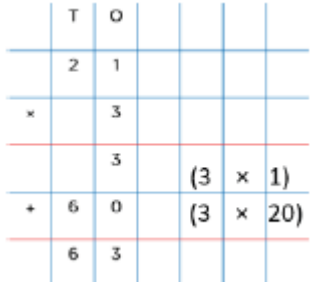
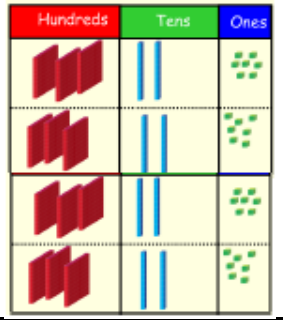
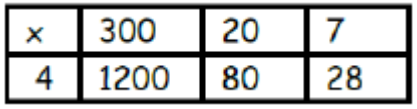
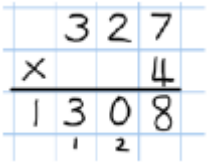
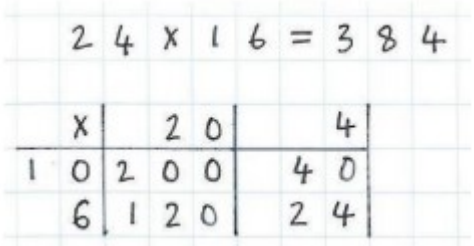
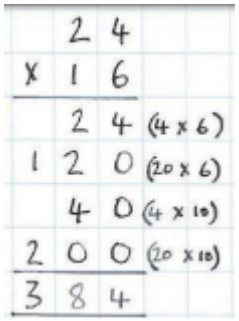
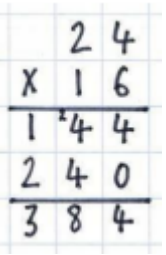
Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Counting in multiples of 2, 3, 4, 5, 10 from 0	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. 	Number lines, counting-sticks and bar models should be used to show representations of counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 4X3 = ____
Multiplication is commutative	Create arrays using counters, cubes and Numicon  Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. 	Use representations of arrays to show different calculations and explore commutativity. 	12 = 3 X 4 12 = 4 X 3 Use an array to write multiplication sentences and reinforce repeated addition 

Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Using the inverse *This should be taught alongside division, so pupils learn how they work alongside each other*			Show all 8 related fact family sentences 2 X 4 = 8 4 X 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 X 4 8 = 4 X 2 2 = 8 ÷4 2 = 8 ÷ 2
Partition to multiply	Use Numicon, Base Ten or Cuisenaire Rods 4 X 15 	Represent the concrete manipulatives Pictorially. 	Show the steps they have taken.  A number line can also be used 
Column Method (multiplying by a single digit number without regrouping)	Use the understanding of partitioning and repeated addition to represent a two-digit number multiplied by a one-digit number.  3 4 x 2  21 x 3	Represent the counters pictorially 	Record what they are doing to show understanding 3 × 23 20 3 3 × 20 = 60 3 × 3 = 9 60 + 9 = 69 23 × 3 69

Multiplication

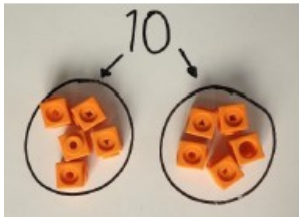
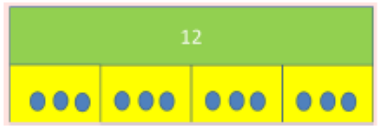
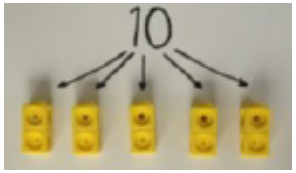
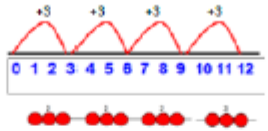
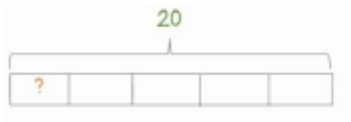
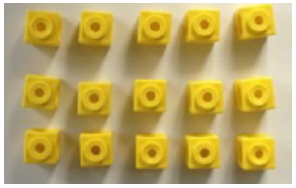
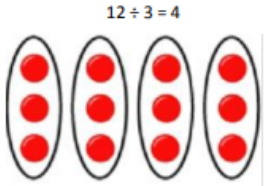
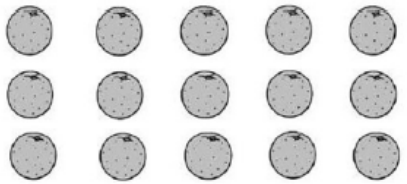
Objective & Strategy	Concrete	Pictorial	Abstract
Column Method of multiplying by a single digit number with regrouping *Moving on to 3 and 4 digit numbers*	Move on to explore multiplication with regrouping when there are 10 or more in a place value c 	Represent the counters pictorially 23 X 6 	 6 x 23 = 23 × 6 138 1 1
Column Multiplication for 3 and 4 digits X 1 This can also be known as the grid method	It is important at this stage that they always multiply the ones first. Children can continue to be supported by place value counters. 		327 x 4 28 80 1200 1308 This will lead to a compact method 
Grid method of multiplying by 2 digit numbers			 

Multiplication

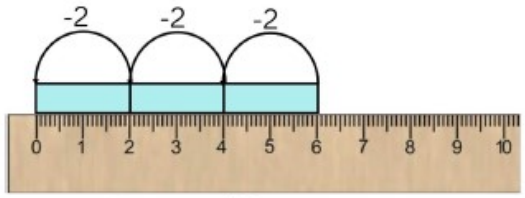
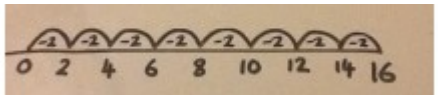
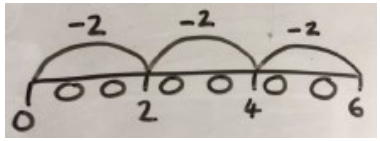
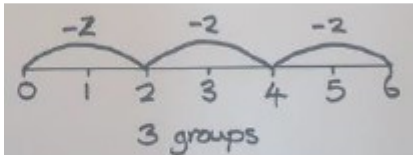
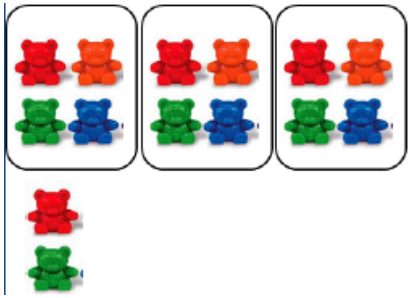
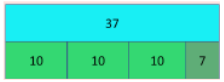
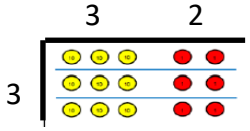
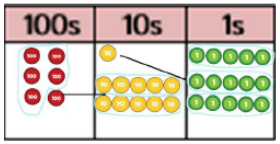
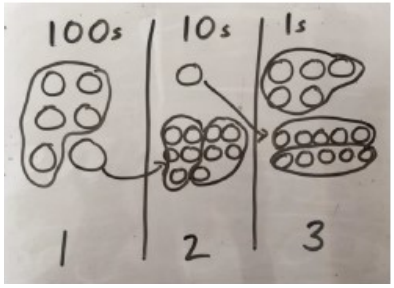
Objective & Strategy	Concrete	Pictorial	Abstract
Expanded Column method of multiplying by 2 or 3 digit numbers	Each value is multiplied and the true value recorded in columns. All values are added to find the Total.	279 x 36 = 10,044 First, multiply through by 6 Then, multiply through by 30 279 x 36 279 x 36 54 (6 x 9) 420 (6 x 70) 1200 (6 x 200) 270 (30 x 9) 2100 (30 x 70) + 6000 (30 x 200) 10,044	124 x 26 744 2480 3224 Answer: 3224
Multiplying decimals up to 2 decimal places by a single digit			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and answer 3.19 x 8 25.52

Division

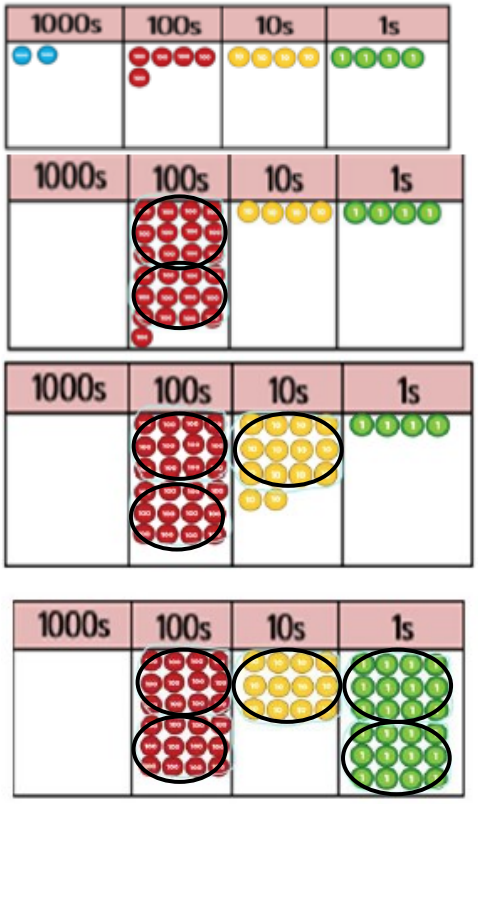
Key Vocabulary: share, group, divide, dived by, half

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	I have 10 cubes, can you share them equally in 2 groups?   	Use pictures or shapes to share quantities.  8 ÷ 2 = 4 Use bar modelling to show and support Understanding 12 ÷4 = 3  	10 shared by 2 is 5 10 ÷ 2 = 5 Encourage them to use their two times table facts
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding  96 ÷ 3 = 32  	Use number lines for grouping. 12 ÷ 3  Think of the bar as a whole split into the number of groups you are dividing by and work out how many would be within the group. 20 ÷ 5 = 5 X ? = 20  	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
Division as arrays	Link division to multiplication by creating an array.   	Draw an array and use lines to split the array into groups. 	Find the inverse of multiplication and division sentences by creating linking number sentences. 7 X 4 = 28 28 ÷ 7 = 4 4 X 7 = 28 28 ÷ 4 = 7

Division

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated subtraction	Use Cuisenaire rods above a ruler 6 ÷ 2  3 groups of 2	Represent subtraction pictorially. 16 ÷ 2 = 8  6 ÷ 2 = 3 	Abstract number line to represent the equal groups that have been subtracted 
Division with remainders	Divide objects between groups and see how much is left over. 14 ÷ 3 = 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders. 	Complete written divisions and show the remainder using r. 
Divide at least 3 digit numbers by 1 digit Short division	Use the place value counters to divide using the bus stop method alongside 96 ÷ 3  Use the place value counters to group 615 ÷ 5 	Represent the place value counters pictorially 615 ÷ 5  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. $4\overline{)872}$ = 218 Move onto divisions with a remainder. $5\overline{)432}$ = 86 r 2 Finally move into decimal places. $4\overline{)5110}$ = 127.75

Division

Objective & Strategy			
Long division	Use place value counters 254 ÷ 12 We can't group 2 thousands into groups of 12 so we regroup them. We can group 24 hundreds into groups of 12 which leaves with 1 hundred. After regrouping the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens. After regrouping the 2 tens, we have 24 ones. We can group 24 ones into groups of 12, which leaves no remainder.		02 12|2544 24 1 021 12|2544 24 14 12 2 0212 12|2544 24 14 12 24 24 0

Division

Objective & Strategy			
Long division with a remainder in any of the place values			

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8	h t o 1 2)2 7 8 -2 0	h t o 1 8 2)2 7 8 -2 ↓ 0 7
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2)2 7 8 -2 0 7	h t o 1 3 2)2 7 8 -2 0 7 - 6 1	h t o 1 3 2)2 7 8 -2 0 7 - 6 1 8
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 -1 8 0	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 -1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.